Homework 7

P6.2.8 Determine V_O in Figure P6.2.8.

Solution: The linear-output current source is transformed to its equivalent voltage source. The mesh current equations are:

Mesh 1: $7I_1 - I_2 - 2I_3 = 80$

Mesh 2: $-I_1 + 7I_2 - 2I_3 = 0$

Mesh 3: $-2I_1 - 2I_2 + 4I_3 = 40$

Solving these equations gives I_1 = 20 A, I_2 = 10 A, and I_3 = 25 A. It follows that $V_O = 4I_2 = 40$ V.

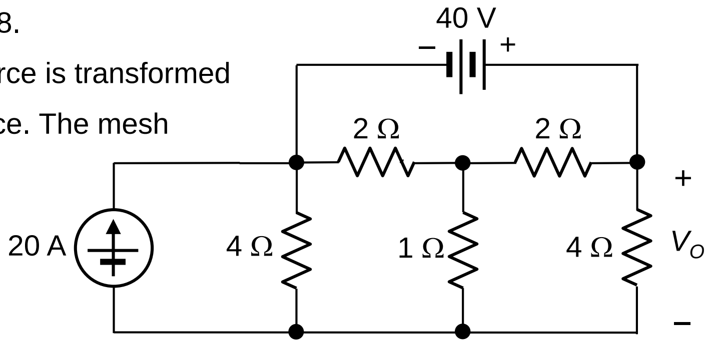

Figure P6.2.8

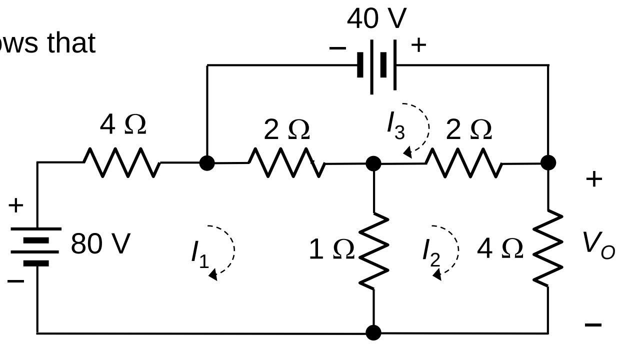

Figure P6.2.8-1

P6.2.12 Determine V_O in Figure P6.2.12.

Solution: The mesh-current equations are:

Mesh 1: $30I_1 - 10I_2 - 20I_3 = 40$

Mesh 2: $-10I_1 + 20I_2 = -20$

Mesh 3: $I_3 = 0.5I_O = 0.5(I_1 - I_2)$

Solving these equations gives I_1 = 2A, I_2 = 0, and I_3 = 1 A. It followshat $V_O = 10I_O = 20$ V.

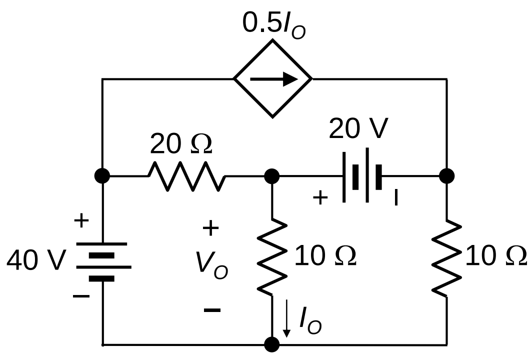

Figure P6.2.12

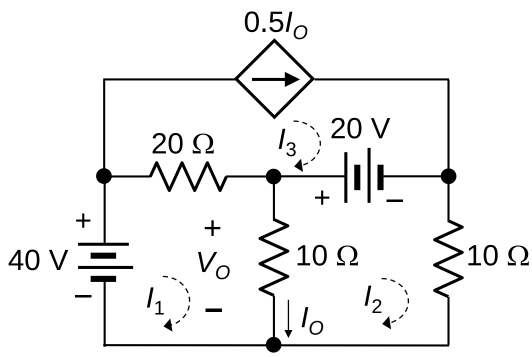

Figure P6.2.12-1

P6.2.22 Determine V_O in Figure P6.2.22, assuming all resistances are 2 Ω.

$I_X = I_3 - I_1 = -0.5$ A and $V_Y = 20I_2 = -10$ V.

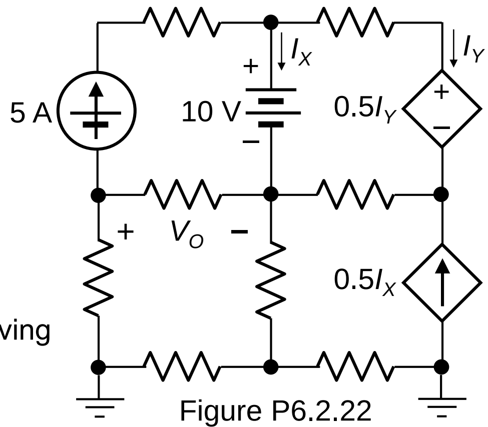

Figure P6.2.22

Solution: The redundant resistor in series with the 5 A current source is removed, and the CCVS is replaced by a 0.5 Ωresistor. Moreover, it is necessary to remove the “hidden mesh” involving the two grounds. To do so, it is noted that the two bottom 2 Ω resistors are in parallel, so they form a 1 Ω resistor in series with the 2 Ω resistor, the circuit becoming as shown.

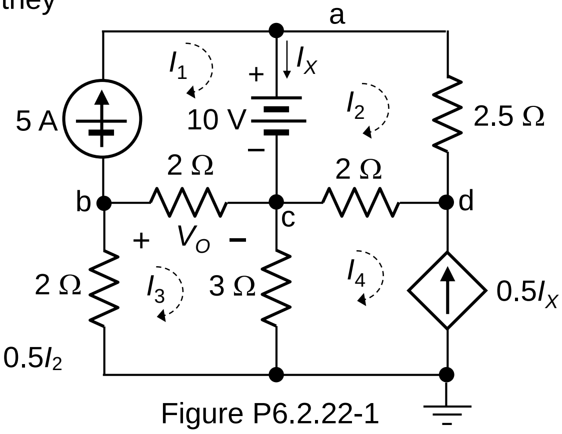

Figure P6.2.22-1

The mesh-current equations are:

Mesh 1: $I_1 = 5$

Mesh 2: $4.5I_2 - 2I_4 = 10$

Mesh 3: $-2I_1 + 7I_3 - 3I_4 = 0$

Mesh 4: $I_4 = -0.5I_X = -0.5(I_1 - I_2)$, or, $0.5I_1 - 0.5I_2 + I_4 = 0$

Solving these equations gives $I_2 = 10/7$ A, $I_3 = 65/98$ A, and $I_4 = -25/14$ A. It follows that $V_O = 2(I_3 - I_1) = -425/49 = -8.67$ V.

P6.2.23 Determine V_X in Figure P6.2.23.

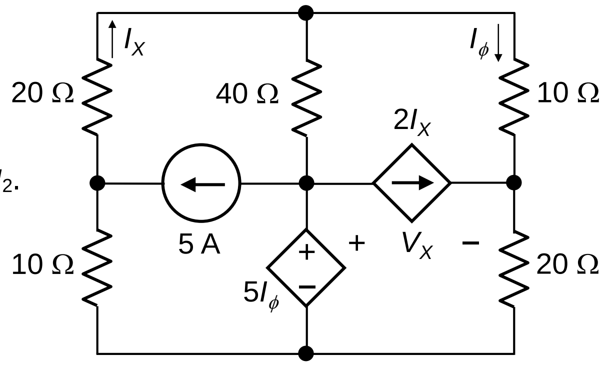

Figure P6.2.23

Solution: The mesh-current equations are:

Mesh 1: $60I_1 - 40I_2 = V_Y$

Mesh 3: $10I_3 = -V_Y - 5I_\phi$, where $I_\phi, = I_2$.

Adding these two equations and substituting for I_ϕ:

$60I_1 - 35I_2 + 10I_3 = 0$

For the 5 A source, $I_1 - I_3 = 5$

Mesh 2: $-40I_1 + 50I_2 = V_X$

Mesh 4: $20I_4 = 5I_\phi - V_X$. Adding these two equations and substituting for I_ϕ:

$-40I_1 + 45I_2 + 20I_4 = 0$

For the dependent current source, $2I_X = I_4 - I_2$, where $I_X = I_1$, or, $2I_1 + I_2 - I_4 = 0$. Solving these equations gives $I_1 =$

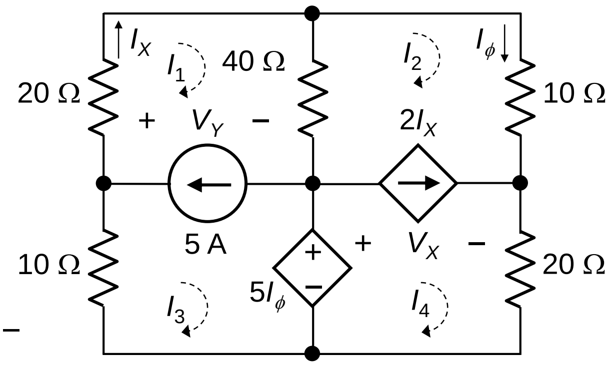

Figure P6.2.23-1

5/7 A, $I_2 = 0$, $I_3 = -30/7$, and $I_4 = 10/7$. It follows that $V_X = 40(I_2 - I_1) + 10I_2 = -40I_1 = -200/7 = -28.57$ V.

P7.1.8 The voltage waveform shown in Figure P7.1.8 is applied to a 1 μF capacitor. Determine the maximum value of the current through the capacitor.

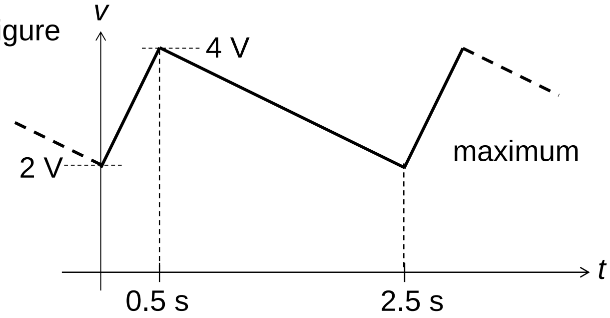

Figure P7.1.8

Solution: The maximum capacitor current occurs on the steeper region of the voltage waveform. $i = Cdv/dt = 10^{-6}\times(2\text{ V}/0.5\text{ s}) = 4\ \mu\text{A}$.

P7.1.10 When the switch is closed in Figure P7.1.10, a current i flows that charges the capacitor. After a sufficiently long time, the capacitor is fully charged to 2 V. Determine, when the capacitor is fully charged: (a) the energy stored in the capacitor; (b) the total energy delivered by the battery, by considering the total charge delivered by the battery.

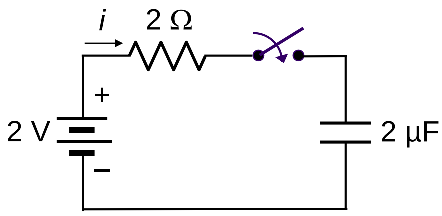

Figure P7.1.10

Soution: (a) When the final voltage is 2 V, the energy stored in the capacitor is $(1/2)\times2\times10^{-6}\times4 = 4\ \mu\text{J}$.

(b) The total energy supplied by the battery is $w(\infty) =$

$$\int_0^\infty 2idt = 2\int_0^\infty idt = 2q(\infty) = 2\times2\times10^{-6}\times2 =$$

8 μJ.

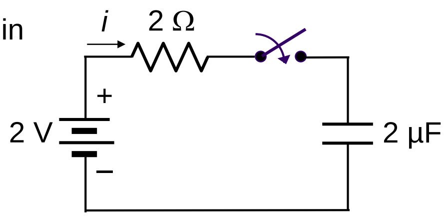

Figure P7.1.10

P7.1.15 The triangular voltage pulse of Figure P7.1.15 is applied to an initially-uncharged 0.1 μF capacitor. Plot as functions of time: (a) the charge on the capacitor; (b) the energy stored in the capacitor, (c) the instantaneous power input to the capacitor.

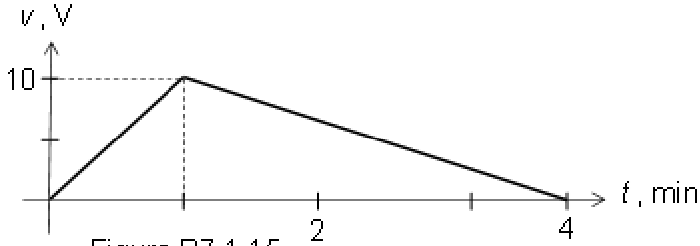

Figure P7.1.15

Solution: $v(t) = \frac{t}{6}$ V, $0 \leq t \leq 60$ s; $v(t) = -\frac{t}{18} + \frac{40}{3}$ V, $60 \leq t \leq 240$ s; $v(t) = 0$, $t \geq 240$ s.

(a) $q(t) = \frac{t}{60}\ \mu$C, $0 \leq t \leq 60$ s; $q(t) = -\frac{t}{180} + \frac{4}{3}\ \mu$C, $60 \leq t \leq 240$ s; $q(t) = 0$, $t \geq 240$ s.

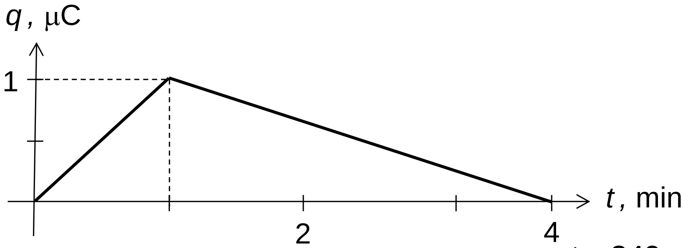

Figure P7.1.15-1

(b) $w(t) = \frac{1}{2}qv = \frac{t^2}{720}\ \mu$J, $0 \leq t \leq 60$ s; $w(t) = \frac{1}{180}\left(\frac{t^2}{36} - \frac{40t}{3} + 1600\right)$ μJ, $60 \leq t \leq 240$ s; $w(t) = 0$, $t \geq 240$ s.

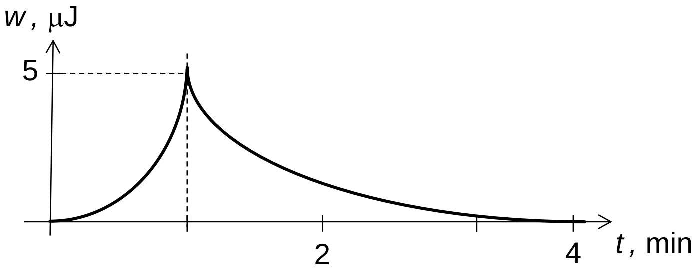

Figure P7.1.15-2

(c) $i(t) = \frac{dq}{dt} = \frac{1}{60}\ \mu$A, $0 < t < 60$ s; $i(t) = -\frac{1}{180}\ \mu$A, $60 < t < 240$ s;

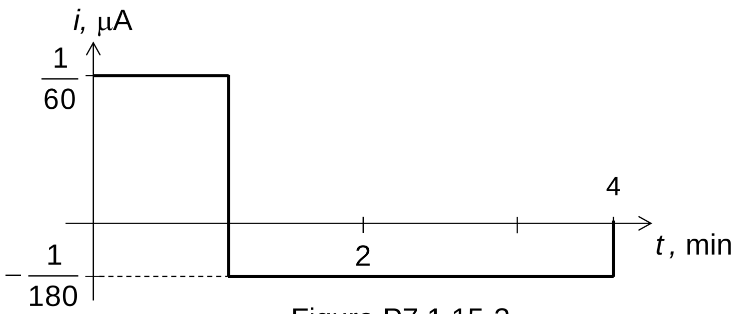

Figure P7.1.15-3

$i(t) = 0,\ t > 240\,\text{s}$, where $\frac{1}{60}$ μA may also be expressed as 1 μC/min.

(d) $p(t) = vi = \frac{t}{360}$ μW,

$0 \le t \le 60\,\text{s}$; $p(t)$ =

$\frac{1}{180}\left(\frac{t}{18} - \frac{40}{3}\right)$ μW,

$60 \le t \le 240\,\text{s}$; $p(t) = 0$,

$t \ge 240\,\text{s}$, where $\frac{1}{6}$ μW may also be expressed as 10 μJ/min.

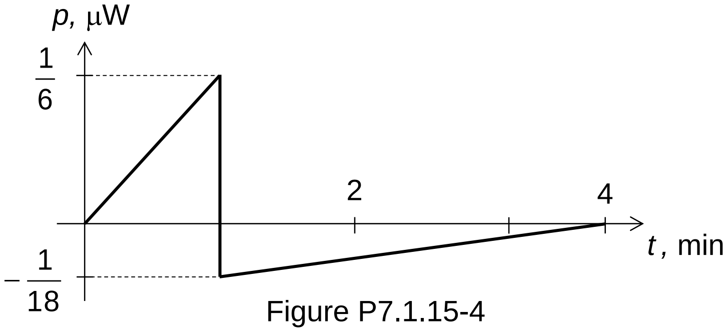

Figure P7.1.15-4

It is seen that $w(t) = \int p\,dt$. Thus $\int_0^t \frac{t}{360}dt = \frac{t^2}{720}$ μJ, $0 \le t \le 60\,\text{s}$. At t = 60 s,

$w(t)$ = 5 μJ. For $60 \le t \le 240\,\text{s}$ $w(t) = \int_{60}^t \frac{1}{180}\left(\frac{t}{18} - \frac{40}{3}\right)dt + 5$

$$= \frac{1}{180}\left(\frac{t^2}{36} - \frac{40t}{3} + 1600\right) \mu\text{J}.$$

P7.1.16 The current waveform of Figure P7.1.16 is applied to an initially-uncharged 0.5 μF capacitor. (a) Derive expressions, as functions of time, for the voltage across the capacitor during the time intervals: $0 \le t \le 10$ μs, $10 \le t \le 40$ μs, $40 \le t \le 60$ μs, $60 \le t \le 80$ μs, and $t > 80$ μs. (b) What is the charge on the capacitor at $t = 10$ μs and at $t = 50$ μs? Check the final value of voltage against the final charge. (c) What is the energy stored in the capacitor at $t = 80$ μs? (d) How do the expressions for the voltage across the capacitor derived in (a) above change it the capacitor was initially charged to 0.5 V?

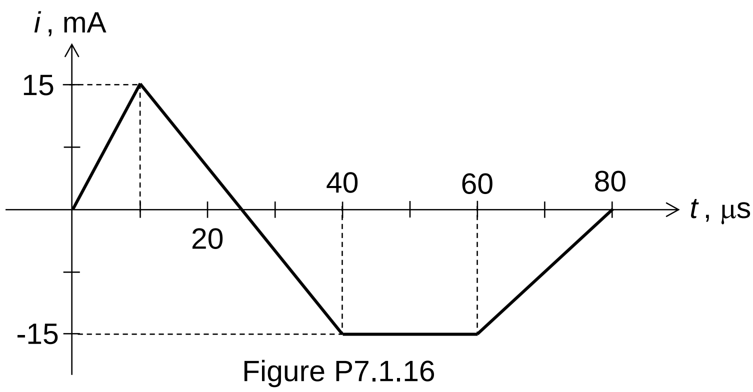

Figure P7.1.16

Solution: $v(t) = \frac{1}{C}\int_{t_0}^{t} i dt + V(t_0)$

(a) $0 \le t \le 10\,\mu$s:

$v(t) = \frac{1}{0.5}\int_0^t 1.5t dt =$ $1.5t^2$ mV, where t is in μs. At t = 10 μs, $v(t)$ = 150 mV.

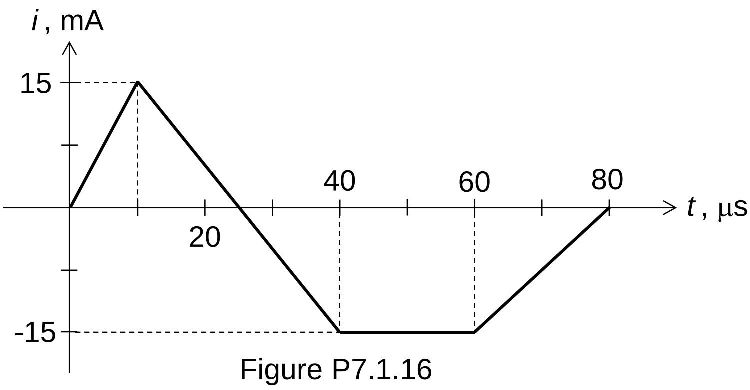

Figure P7.1.16

$10 \le t \le 40\,\mu$s: $v(t) = \frac{1}{0.5}\int_{10}^{t}(-t + 25)dt + 150$

$= \left[-t^2 + 50t\right]_{10}^{t} + 150 = -t^2 + 50t - 250$ mV. At t = 40 μs, $v(t)$ = 150 mV.

$40 \le t \le 60\,\mu$s: $v(t) = \frac{1}{0.5}\int_{40}^{t} -15dt + 150 = \left[-30t\right]_{40}^{t}$ +150 = –30t + 1350 mV.

At t = 60 μs, $v(t)$ = -450 mV.

$60 \le t \le 80\,\mu$s: $v(t) = \frac{1}{0.5}\int_{60}^{t}(0.75t - 60)dt - 450$ $= \left[0.75t^2 - 120t\right]_{60}^{t} - 450 =$

$0.75t^2 - 120t + 4050$ mV. At t = 80 μs, $v(t)$ = -750 mV.

$t \ge 80\,\mu$s: v = –750 mV.

Check: Total area = 0.5×15×10 + 0.5×15×15 – 0.5×15×15 – 15×20 – 0.5×15×20 = –375 nC. Hence v = $\frac{-375}{0.5} = -750$ mV.

(b) At $t = 10\,\mu$s, $v(t) = 150$ mV, so $q = CV = 75$ nC.

At $t = 50\,\mu$s, $v(t) = -150$ mV, so $q = CV = -75$ nC.

(c) At $t = 80\,\mu$s, $v(t)$ = –750 mV, so $w = \frac{1}{2}Cv^2 = 0.14\,\mu$J.

(d) All the expressions derived above for the voltage are increased by 0.5 V.